Enrollment No:	Exam Seat No:
----------------	---------------

# **C.U.SHAH UNIVERSITY**Summer Examination-2017

**Subject Name: Discrete Mathematics** 

Subject Code: 4TE04DSM1 Branch: B.Tech (CE,IT)

Semester: 4 Date: 03/05/2017 Time: 02:00 To 05:00 Marks: 70

#### **Instructions:**

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1	Attempt the following questions:	(14)
a)	Find the least and greatest element in the POSET $\langle N, D \rangle$ , if they exist.	(01)
<b>b</b> )	Define: Equivalence relation, Comparable element.	(02)
<b>c</b> )	State Pigeonhole principle.	(01)
d)	Prove that $(ab+ab')a'b'=0$ .	(02)
e)	Find the atom and anti-atom of $\langle P(X), \subseteq \rangle$ .	(01)
f)	State Handshaking theorem and define cycle.	(02)
g)	$(Z_{10}, +_{10})$ is cyclic group True or False?	(01)
h)	Define: Difference of two fuzzy sets.	(02)
i)	In how many ways can the letters of the word MONDAY be arranged? How many of	(02)
1)	them begin with M and end with Y?	(02)

### Attempt any four questions from Q-2 to Q-8

# Q-2 Attempt all questions. (14)

- a)  $\langle L, *, \oplus \rangle$  is a lattice an algebraic system then there exist an order relation  $\leq$  on L such that  $\langle L, \leq \rangle$  is a lattice as a poset. Where  $a * b = \text{glb}\{a, b\}$ ,  $a \oplus b = \text{lub}\{a, b\}$  for  $\forall a, b \in L$ .
- **b)** For a lattice  $\langle S_{60}, D \rangle$ , answer the following questions: (05)
  - i) Find cover of each element and draw the Hasse diagram.
  - ii) Find lower bound, upper bound, greatest lower bound, least upper bound of  $A = \{6,12\}$ .
- c)  $\langle L, *, \oplus \rangle$  is a modular lattice if and only if  $(a*b) \oplus (a*c) = a*(b \oplus (a*c))$ . (04)



## Q-3 Attempt all questions

**(14)** 

- a) Show that  $\{1,5,7,11\}$  is a subgroup of  $(Z_{12}^*,\times_{12})$ , where  $\times_{12}$  is multiplication modulo 12. (05)
- **b)** Prove that  $\langle P(X), \subseteq \rangle$  is a complemented lattice and also draw the Hasse diagram of it, where  $X = \{1, 2, 3\}$ .
- c) Show that w is a valid conclusion from the premises  $r \lor s$ ,  $\sim t \Rightarrow \sim r$ ,  $\sim s$ ,  $t \Rightarrow w$ . (04)

### Q-4 Attempt all questions

**(14)** 

- a) Let  $\langle L, *, \oplus, ', 0, 1 \rangle$  be a complemented lattice and for any  $a, b \in L$  then prove that  $a \le b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \le a' \Leftrightarrow a' \oplus b = 1.$  (05)
- **b)** Let  $E = \{0,1,2,3\}$ ,  $A = \{(0,0.2),(1,0.4)(2,0.6),(3,0.7)\}$ , (05)  $B = \{(0,0.4),(1,0.6)(2,0.8),(3,0.8)\}$  then find the following:
  - 1)  $\underset{\sim}{A} \cup \underset{\sim}{B}$  2)  $\underset{\sim}{A} \cdot \underset{\sim}{B}$  3)  $\underset{\sim}{A} + \underset{\sim}{B}$  4)  $\underset{\sim}{A} \underset{\sim}{B}$  5)  $\underset{\sim}{A}'$
- c) Obtain the sum of product canonical form of the Boolean expression in three variables  $\alpha(x, y, z) = (x + y)(x + z') + y + z'$ .

## Q-5 Attempt all questions

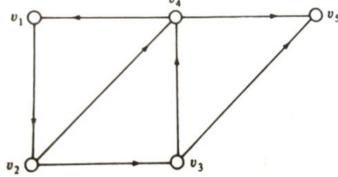
(14)

- a) State and prove Stone's representation theorem. (10)
- b) State Distributive law for fuzzy subsets and prove any one. (04)

# Q-6 Attempt all questions

**(14)** 

- a) i) Draw the graph represented by given adjacency matrix  $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$  (05)
  - ii) Write the adjacency matrix from the given digraph.



**b)** Show that the set  $Q \setminus \{1\}$  is an abelian group with respect to the binary operation a\*b=a+b-ab, for all  $a,b \in G$ .

Page 2 of 3



- c) Solve the recurrence relation  $a_n = 5a_{n-1} 6a_{n-2}$ ,  $n \ge 2$ ;  $a_0 = 1$ ,  $a_1 = 2$ . (04)
- Q-7 Attempt all questions. (14)
  - a) Define: Pseudo graph, Cycle, Reachable set, Node base, Level of vertex (05)
  - b) Prove that  $(Z_5, +_5)$  is a group. Moreover check that it is cyclic or not, if it is cyclic (05) then find generators.
  - c) By using mathematical induction prove that  $1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ . (04)
- Q-8 Attempt all questions. (14)
  - a) Draw a directed tree from following and also find the representation of binary tree.  $(v_0(v_1((v_2)(v_3)(v_4(v_5)(v_6))))(v_7((v_8((v_9)(v_{10})(v_{11})))(v_{12}))))$
  - **b**) State and prove the Lagrange's theorem. (05)
  - c) Do as directed: (04)
    - 1) Translate each of the statement into symbols, using quantifiers, predicate symbols.
      - i) "Every student either can speak Tamil or knows C++."
      - ii) "Some men are genius."
    - 2) Translate the following in your own words.

A(x): x is a whale, B(x): x is a fish, C(x): x lives in water.

i) 
$$(\exists x)(B(x) \land \sim A(x))$$

ii) 
$$(\forall x)(A(x)\lor C(x))\Rightarrow B(x)$$